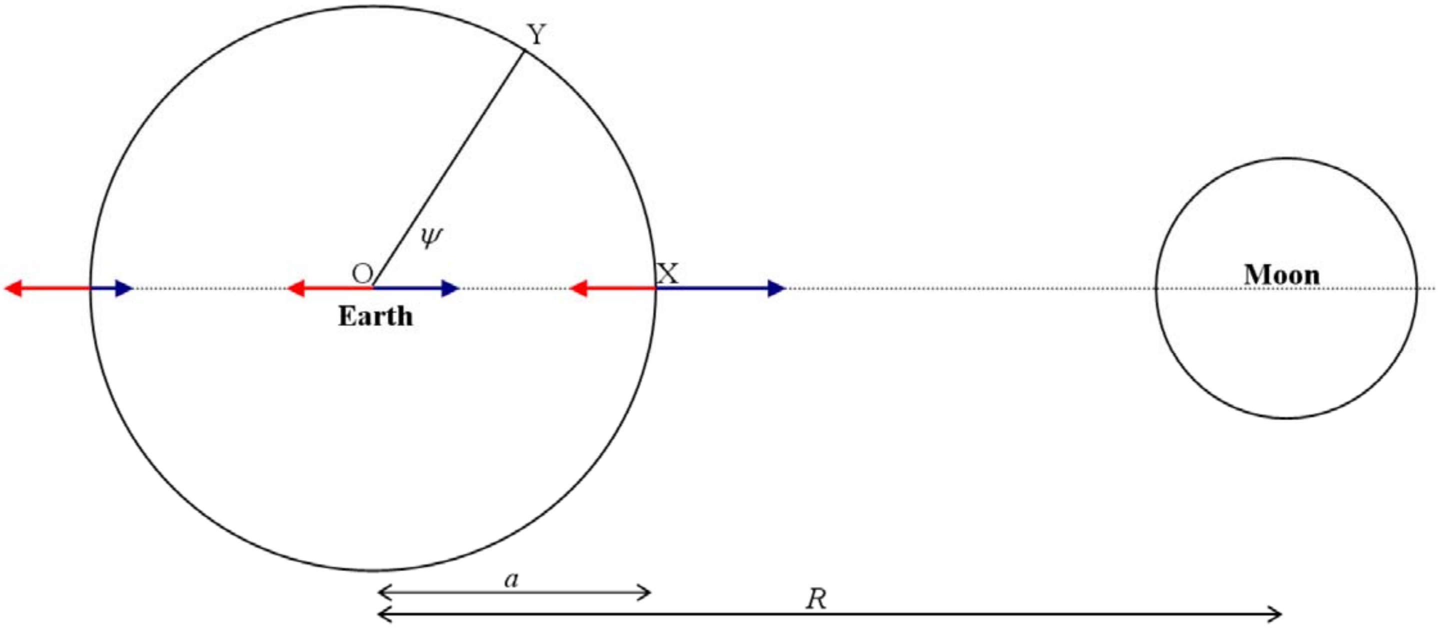
The Tide Producing Force



The gravitational force between two bodies is given by:

$$F_g = \frac{GM_1M_2}{R^2}$$

Consider the point X. The gravitational attraction of the moon at X is greater than that at the Earth's centre as it is nearer by the distance of the Earth's radius (a). Because the gravitational force exerted by the Moon on a point at the Earth's centre is exactly equal and opposite to the centrifugal force there, the tide producing force at the centre of the earth is zero. Now as the centrifugal force is equal at all points on Earth, and at Earth's centre is equal to the gravitational force exerted by the Moon, it follows that:

$$\begin{aligned} TPF_X &= \frac{GM_1M_2}{(R-a)^2} - \frac{GM_1M_2}{R^2} \\ &= \frac{R^2(GM_1M_2) - (R-a)^2(GM_1M_2)}{R^2(R-a)^2} \\ &= \frac{GM_1M_2(R^2-(R-a)^2)}{R^2(R-a)^2} \\ &= \frac{GM_1M_2(R^2-R^2+2Ra-a^2)}{R^2(R-a)^2} \\ &= \frac{GM_1M_2a(2R-a)}{R^2(R-a)^2} \end{aligned}$$

But a is very small compared to R, so $(2R-a)$ can be approximated to $2R$, and $(R-a)^2$ to R^2, giving the approximation:

$$TPE \approx \frac{GM_1M_2 2a}{R^3}$$

For points that do not lie on the line joining the centre of the Moon and the Earth, the equation would need to be:

$$F_g = \frac{GM_1M_2}{(R - a\cos\psi)^2}$$